수 학 Ⅲ

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-- 머 리 말 --

20년을 넘게 대학에서 수학을 강의하면서 영재들을 위한 수학 문제 창작에 깊은 관심을 가졌다. 우연히 수학문제 풀이 방법에 숙련된 일반 학생과 영재 학생이 푸는 방식이 전혀 다르다는 것을 보고서 일반 학생들이 영재 학생으로 성장해 가게 하는 수학을 만들어 보게 되었다. 일반 고등학교에서 수학Ⅱ 및 미분적분의 개념을 정확히 이해한 학생이면 모두 풀 수 있게 문제들을 구성하였다. 모든 문제들을 숙련된 풀이 방식의 접근이 아니라 창의적인 생각으로 각각 다르게 접근하여 풀어보면 수학의 참 맛을 느낄 수 있을 것으로 생각된다.

일반 학생들은 첫 문제를 보고, “풀 수 있겠다. 또는 풀 수 없겠다.” 고민하는 시간들을 측정해 본 결과 30초를 넘지 않았다. 대부분 1분 정도 지나면 포기해 버린 모습에서 영재 학생들과 분명히 다른 면모를 알 수 있었다. 보통 영재 학생들은 어떤 문제를 해결하기 위해 10시간 넘게 고민하는 것을 종종 본다. 10시간 넘게 고민하면서 해결한 문제들은 말로 표현할 수 없는 쾌감을 줄 것이며 학생의 뇌리에 영원히 각인될 것이다. 이런 훈련이 쌓인다면 여러분도 종종 10시간 넘게 고민하는 모습들을 볼 수 있게 될 것이라 생각되며 신비스러운 수학 세계를 경험하게 될 것이다.

역사적으로 지혜를 겸비한 지도자들의 공통점은 생각하는 힘이 일반인보다 월등하며 쉽게 창의적인 생각을 이끌어 낸다는 점이다. 이러한 능력은 훈련을 통해서 습득할 수 있는데 논리적인 사고에 기초를 둔 수학이론들을 배우게 하여 지도자를 양성했던 기록들이 있다.

지혜를 겸비한 인재로 성장하여 여러분 모두 각각의 분야에서 지도자로 발전하길 기원하는 마음으로 저자는 이 책을 만들게 되었다.

『 생각하는 지혜는 마음으로 풍요로운 삶을 느낄 수 있다. 』

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-- 목 차 --

A. 다음 부등식을 증명하여라.

[1]. $a^t b^{1-t} \leq ta+(1-t)b$, $(a,b>0,\ 0 \leq t \leq 1)$

[2]. $\left(1+\frac{1}{n}\right)^n < \left(1+\frac{1}{n+1}\right)^{n+1}$, $(n \in N)$

[3]. $(x^n y^m)^{\frac{1}{n+m}} \leq \frac{nx+my}{n+m}$, $(x,y>0,\ n,m \geq 0)$

[4]. $\ln x \leq 2(\sqrt{x}-1)$, $(x \geq 1)$

[5]. $1+h < e^h < 1+he^h$, $(0<h)$

[6]. $\sqrt{\frac{1-x}{1+x}} < \frac{\ln(1+x)}{\sin^{-1}x} < 1$, $(0<x<1)$

[7]. $(a_1 a_2 \bullet \bullet \bullet a_n)^{\frac{1}{n}} \leq \frac{a_1+a_2+\bullet \bullet \bullet + a_n}{n}$, $(a_{1,} \dots > 0)$

[8]. $(a_1 a_2 \bullet \bullet a_n)^{\frac{1}{n}} \geq \frac{n}{\left(\frac{1}{a_1}+\frac{1}{a_2}+\bullet \bullet + \frac{1}{a_n}\right)}$, $(a_{1,} \dots > 0)$

[9]. $\frac{1}{2}+\frac{1}{3}+\bullet \bullet + \frac{1}{n} < \ln n < 1+\frac{1}{2}+\bullet \bullet + \frac{1}{n-1}$, $(2 \leq n)$

[10]. $\dfrac{x}{1+x} < \ln(1+x) < x,\ (x>0)$

[11]. $\tan^{-1}\sqrt{1-x} < \dfrac{\pi - x}{4},\ (0<x\le 1)$

[12]. $x < \sin^{-1}x < \dfrac{x}{\sqrt{1-x^2}},\ (0<x<1)$

[13]. $\dfrac{1}{x+1} < \ln\left(1+\dfrac{1}{x}\right) < \dfrac{1}{x},\ (0<x)$

[14]. $a_n = 1 + \dfrac{1}{2} + \cdot\ \cdot + \dfrac{1}{n} - \ln n \Rightarrow a_{n+1} < a_n,\ (2 \le n)$

[15]. $\dfrac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \dfrac{b-a}{1+a^2},\ (a<b)$

[16]. $1 \le \displaystyle\int_0^1 \sqrt{1+x^4}\,dx \le \dfrac{4}{3}$

[17]. $2 < e < 3$

[18]. $a^b > b^a,\ (0<b<a\le e)$

[19]. $b^a < a^b,\ (e \le a < b)$

[20]. $e^x < \dfrac{1}{1-x},\ (0<x<1)$

[21]. $\ln\dfrac{x+1}{x} < \dfrac{1}{x} < \ln\dfrac{x}{x-1},\ (x>1)$

[22]. $\sqrt{1+x} < 1 + \frac{x}{2}$, $(x > 0)$

[23]. $\frac{1}{2} + \frac{1}{4} + \cdot \cdot + \frac{1}{2^{n-1}} < 1,\ (n > 1)$

[24]. $(a_1 b_1 + \cdot \cdot + a_n b_n)^2 \le (a_1 + \cdot \cdot + a_n)^2 (b_1 + \cdot \cdot + b_n)^2, \left(a_i, b_j \in R^+\right)$

[25]. $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$

[26]. $\left(1 - \frac{a}{b}\right) < \ln\frac{b}{a} < \left(\frac{b}{a} - 1\right),\ (0 < a < b)$

[27]. $\left|\int_0^1 \frac{\cos nx}{x+1} dx\right| \le \ln 2$

[28]. $\left|\int_0^{2\pi} \frac{\sin nx}{x^2 + n^2} dx\right| \le \frac{2\pi}{n^2}$

[29]. $|\sin x - \sin y| \le |x - y|$

[30]. $\frac{x}{1+x^2} < \tan^{-1} x < x,\ (0 < x)$

[31]. $ab + cd \le 1,\ (a^2 + b^2 = 1, c^2 + d^2 = 1)$

[32]. $ac + bd \le 1,\ (a^2 + b^2 = 1,\ c^2 + d^2 = 1)$

[33]. $|\sin nx| \le n\sin x,\ \ (0 < x < \pi)$

[34]. $x^{n+1} + \dfrac{1}{x^{n+1}} \ge x^n + \dfrac{1}{x^n}\ ,\ (x > 0)$

[35]. $\left|\displaystyle\int_1^{\sqrt{3}} \frac{e^{-x}\sin x}{x^2+1}dx\right| \le \dfrac{\pi}{12e}$

[36]. $\dfrac{26}{3} \le \displaystyle\int_1^3 \sqrt{1+x^4}\,dx \le \frac{26}{3}\sqrt{2}\,,\ (x \ge 1)$

[37]. $\ln x \le x-1,\ \ (x \ge 1)$

[38]. $1 \le \displaystyle\int_0^1 \sqrt{1+x^4}\,dx \le \sqrt{2}\,,\ (0 \le x \le 1)$

[39]. $e^x > \dfrac{x^2}{2}+x+1,\ \ (x > 0)$

[40]. $\sin x \ge x - \dfrac{x^3}{6}\,,\ (x \ge 0)$

[41]. $\cos x \le 1 - \dfrac{x^2}{\pi}\ ,\ \ (0 \le x \le \dfrac{\pi}{2})$

[42]. $\tan x > x + \dfrac{x^3}{3}\ ,\ \ \left(0 < x < \dfrac{\pi}{2}\right)$

[43]. $\cosh x > 1 + \dfrac{x^2}{2}$

[44]. $\cos x \leq 1 - \dfrac{x^2}{2} + \dfrac{x^3}{6}$, $(x \geq 0)$

[45]. $x^a - 1 \geq a(x-1)$, $(a \in N,\ x > 0)$

[46]. $(x+1)^a \geq ax + 1$, $(x > -1,\ a > 1)$

[47]. $\dfrac{\pi}{4} < \displaystyle\sum_{n=1}^{\infty} \dfrac{1}{1+n^2} < \dfrac{1}{2} + \dfrac{\pi}{4}$

[48]. $\dfrac{2}{3} n^{\frac{3}{2}} + \dfrac{1}{3} \leq \displaystyle\sum_{k=1}^{n} \sqrt{k} \leq \dfrac{2}{3} n^{\frac{3}{2}} + \sqrt{n} - \dfrac{2}{3}$

[49]. $\left| \displaystyle\int_0^1 \dfrac{\cos nx}{1+x^2}\, dx \right| \leq \dfrac{\pi}{4}$

[50]. $\left(\displaystyle\int_0^{1/2} \dfrac{\cos \pi x}{\sqrt{1+x^2}}\, dx \right)^2 \leq \dfrac{1}{4} \tan^{-1}\left(\dfrac{1}{2}\right)$

[51]. $(a\cos\theta + b\sin\theta)^2 \leq a^2 + b^2$

[52]. $\left(\displaystyle\int_0^{\infty} \dfrac{\left(\sqrt{\cos \alpha x} + \sqrt{x \sin \alpha x}\right)^2}{1+x^2}\, dx \right)^{1/2} \leq \sqrt{\dfrac{2\pi}{e^{\alpha}}}$

[53]. $\dfrac{1}{n+1} \leq \displaystyle\int_0^1 (1-x^2)^n\, dx$

[54]. $\left(1 + \dfrac{1}{n+1}\right)^n \left[1 + \dfrac{1}{n+1} + \dfrac{1}{n}\right] < \left(1 + \dfrac{1}{n}\right)^{n+1}$, $n \in N$

[55]. $\dfrac{2\cdot 4\cdot\cdot(2n)}{3\cdot 5\cdot\cdot(2n+1)} < \dfrac{1\cdot 3\cdot\cdot(2n-1)}{2\cdot 4\cdot\cdot(2n)}\cdot\dfrac{\pi}{2} < \dfrac{2\cdot 4\cdot\cdot(2n-2)}{3\cdot 5\cdot\cdot(2n-1)}$

[56]. $\sqrt{(a_1+b_1)^2+\cdot\cdot+(a_n+b_n)^2} \le \sqrt{a_1^2+\cdot\cdot+a_n^2} + \sqrt{b_1^2+\cdot\cdot+b_n^2}$

[57]. $n^2 \le (a_1+\cdot\ \cdot+a_n)\left(\dfrac{1}{a_n}+\cdot\ \cdot+\dfrac{1}{a_n}\right), (a_i>0)$

[58]. $\dfrac{a_1+\cdot\ \cdot+a_n}{\sqrt{n}} \le \sqrt{a_1^2+\cdot\ \cdot+a_n^2},\ (a_i>0)$

[59]. $x^a \le ax+(1-a),\ \ (0<a<1,\ x\ge 0)$

[60]. $a^{\frac{1}{n}}-b^{\frac{1}{n}} < (a-b)^{\frac{1}{n}}, (a>b>0,\ n\ge 2)$

[61]. $1-\dfrac{x^2}{2} \le \cos x$

[62]. $1-\dfrac{1}{e} \le \displaystyle\int_0^1 e^{-x^2}dx \le 1$

[63]. $\left|(1+x)^{\frac{1}{3}}-\left(1+\dfrac{x}{3}-\dfrac{x^2}{9}\right)\right| < \dfrac{5}{81}x^3, (x>0)$

[64]. $\left|\ln(1+x)-\left(x-\dfrac{x^2}{2}+\cdot\ \cdot+(-1)^{n-1}\dfrac{x^n}{n}\right)\right| < \dfrac{x^{n+1}}{n+1}, (0<x\le 1)$

[65]. $e^{\pi} > \pi^{e}$

[66]. $$(\sin x)^{\sin x} < (\cos x)^{\cos x}\ ,\ \left(0 < x < \frac{\pi}{4}\right)$$

[67]. $$p+q=1, p,q \in R^{+}, f(x) = \sqrt{x} \Rightarrow f(px+qy) \geq pf(x)+qf(y)$$

[68]. $$\left(\frac{n+1}{n}\right)^{n(n-1)} < \left(\frac{n^n}{n!}\right)^2$$

[69]. $$\sum_{i=0}^{\infty} \frac{1}{(i+j+1)\sqrt{i+\frac{1}{2}}} < \frac{\pi}{\sqrt{j+\frac{1}{2}}}$$

[70]. $$x - \frac{x^2}{2} + \cdot\cdot\cdot - \frac{x^{2k}}{2k} < \ln(1+x) < x - \frac{x^2}{2} + \cdot\cdot\cdot + \frac{x^{2k+1}}{2k+1}, (0 < x \leq 1)$$

[71]. $$0 \leq 3f\left(\frac{x}{3}\right) \leq 2f(x) \leq 2 \Rightarrow f(x) \leq x^{\frac{\ln 3}{\ln 2}-1}, (0 \leq x \leq 1)$$

[72]. $$e^{\int_0^1 \sin\sqrt{x}\,dx} < \int_0^1 e^{\sin\sqrt{x}}\,dx$$

[73]. $$f(0)=0, f(1)=1, f'(x) \geq 0 \Rightarrow \sqrt{2} \leq \int_0^1 \sqrt{1+f'(x)^2}\,dx \leq 2$$

[74]. $$e \leq a \leq b, 0 \leq p \leq q, \frac{p}{q} \leq \frac{a}{b} \Rightarrow b^p \leq a^q$$

[75]. $$(a_1 a_2 \cdot\cdot a_n)^{\frac{1}{n}} \leq (c_1 c_2 \cdot\cdot c_n)^{-\frac{1}{n}} \left(\frac{1}{n}\right) \sum_{k=1}^{n} a_k c_{k,}\ (a_{i,} c_i > 0)$$

[76]. $\sum_{n=1}^{\infty}\left(a_1 a_2 \bullet \bullet \bullet a_n\right)^{\frac{1}{n}} \leq e\sum_{n=1}^{\infty} a_n$

[77]. $f(0)=0, f'(x)>1\,(0<x<1) \Rightarrow \left(\int_0^1 f(x)dx\right)^2 \leq \int_0^1 f(x)^3 dx$

[78]. $\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{6}\right) \bullet \bullet \bullet \left(\frac{2n-1}{2n}\right) < \frac{1}{\sqrt{2n}}$

[79]. $\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{6}\right) \bullet \bullet \bullet \left(\frac{2n-1}{2n}\right) < \frac{1}{\sqrt{3n+1}}$

[80]. $(1+1)\left(1+\frac{1}{2^3}\right)\left(1+\frac{1}{3^3}\right) \bullet \bullet \bullet \left(1+\frac{1}{n^3}\right) < 3-\frac{1}{n}$

[81]. $1+\frac{1}{2^2}+\frac{1}{3^2}+ \bullet \bullet \bullet + \frac{1}{n^2} < 2-\frac{1}{n}$

[82]. $\frac{a}{\sqrt{a^2+8bc}}+\frac{b}{\sqrt{b^2+8ca}}+\frac{c}{\sqrt{c^2+8ab}} \geq 1, (a,b,c>0)$

[83]. $(a-x)^6-3a(a-x)^5+\frac{5}{2}a^2(a-x)^4-\frac{1}{2}a^4(a-x)^2<0, (0<x<a)$

[84]. $f'(x)=\frac{1}{x^2+f(x)^2}, f(1)=1 \Rightarrow f(x)<1+\frac{\pi}{4}, (1 \leq x)$

[85]. $\frac{a}{b} \in (0,1), \frac{a}{b} \in Q \Rightarrow \frac{1}{4b^2} < \left|\frac{a}{b}-\frac{1}{\sqrt{2}}\right|$

[86]. **세근** $\alpha<\beta<\gamma\left(x^3+ax^2+bx+c=0\right)\Rightarrow\sqrt{a^2-3b}<\gamma-\alpha\le\frac{2\sqrt{a^2-3b}}{\sqrt{3}}$

[87]. $\frac{2}{3}n^{\frac{3}{2}}<\sum_{k=1}^{n}\sqrt{k}<\frac{2}{3}n^{\frac{3}{2}}+\frac{\sqrt{n}}{2}$

[88]. **삼각형 세변:** $a,b,c\Rightarrow a^3+b^3+c^3+abc<\frac{2(a+b+c)\left(a^2+b^2+c^2\right)}{3}$

[89]. **감소** $f:[0,1]\rightarrow R^+\Rightarrow\int_0^1 f(x)dx\int_0^1 xf(x)^2dx\le\int_0^1 xf(x)dx\int_0^1 f(x)^2dx$

[90]. $\frac{3n+1}{2(n+1)}<\sum_{k=1}^{n}\left(\frac{k}{n}\right)^n<2,\ (1<n)$

[91]. $n\sqrt[n]{1+n}-n<\sum_{k=1}^{n}\frac{1}{k}<n-\frac{n-1}{\sqrt[n-1]{n}},\ (2<n)$

[92]. $\frac{1}{e}\left(1-\frac{1}{n}\right)<\left(1-\frac{1}{n}\right)^n<\frac{1}{e}\left(1-\frac{1}{2n}\right),\ (n>1)$

[93]. $\frac{xyz}{x^3+y^3+xyz}+\frac{xyz}{y^3+z^3+xyz}+\frac{xyz}{z^3+x^3+xyz}\le 1,\ (x,y,z>0)$

[94]. $c=\frac{a^{a+1}+b^{b+1}}{a^a+b^b}\Rightarrow a^a+b^b\le c^a+c^b,\ (a,b\in N)$

[95]. $a=\frac{x+y+z}{3}\Rightarrow(xyz)^a\le x^xy^yz^z,\ (x,y,z>0)$

[96]. $2m \le n \Rightarrow \dfrac{(n-m)!}{m!} \le \left(\dfrac{n}{2}+\dfrac{1}{2}\right)^{n-2m}, (n,m\in N)$

[97]. $(1+a+a^2)^2 < 3(1+a^2+a^4), (a \ne 1)$

[98]. $1-\dfrac{1}{n} < \int_0^1 \dfrac{1}{1+x^n}dx, (n\in N)$

[99]. $1-\dfrac{1}{a} \le n(\sqrt[n]{a}-1) \le a-1, (n\in N)$

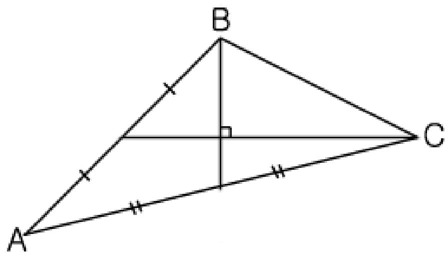
[100]. $ab^{n-1}+bc^{n-1}+ca^{n-1} \le a^n+b^n+c^n, (a,b,c \ge 0)$

[101]. f : 증가, g : 감소 $\Rightarrow \int_0^1 f(x)g(x)dx \le \int_0^1 f(x)g(1-x)dx$

[102]. $(a+b-c)(b+c-a)(c+a-b) \le abc, (a,b,c \ge 0)$

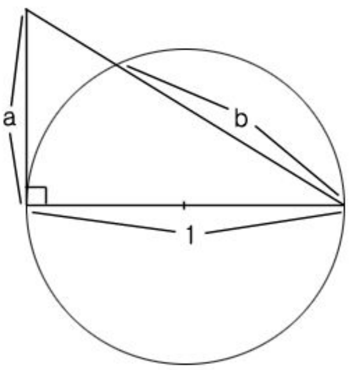
[103]. $\dfrac{(a-b)^2}{8a} < \dfrac{a+b}{2} - \sqrt{ab} < \dfrac{(a-b)^2}{8b}, (a > b > 0)$

[104]. $\cot B + \cot C \ge \dfrac{2}{3}$



[105]. $abc = 1, \dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c} < a+b+c, 0 < c \le b \le a \Rightarrow b < 1 < a$

[106]. $$\frac{1}{a^2+\frac{1}{2}} < \frac{b}{a} < \frac{1}{a^2}$$



[107]. $x+y+z=3 \Rightarrow xy+yz+zx \le \sqrt{x}+\sqrt{y}+\sqrt{z}$

[108]. $\triangle ABC \Rightarrow \sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) < \frac{1}{4}$

[109]. $a^2+b^2+c^2=1 \Rightarrow -\frac{1}{2} \le ab+bc+ca \le 1$

[110]. $n^n < n! \cdot n!,\ (n>2)$

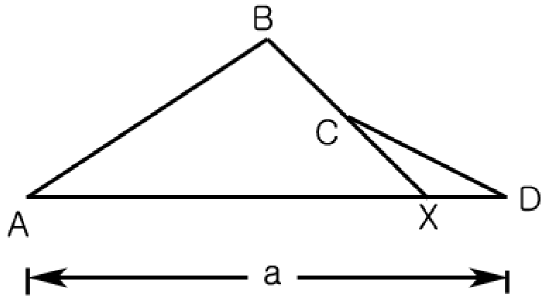
[111]. **예각삼각형**: $A<B<C \Rightarrow \sin 2C < \sin 2B < \sin 2A$

[112]. $\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x > 0,\ (0<x<\pi)$

[113]. $\frac{1}{3^3}+\frac{1}{4^3}+\cdots+\frac{1}{n^3} < \frac{1}{12}$

[114]. $\sqrt[n]{a_1a_2\cdots a_n}+\sqrt[n]{b_1b_2\cdots b_n} \le \sqrt[n]{(a_1+b_1)(a_2+b_2)\cdots(a_n+b_n)}$

[115]. $\angle ABX > 120^{\circ} \Rightarrow \overline{AB} + \overline{BC} + \overline{CD} \leq \dfrac{2a}{\sqrt{3}}$



[116]. $2^{\frac{1}{2}} 4^{\frac{1}{4}} 8^{\frac{1}{8}} \cdots (2^n)^{\frac{1}{2^n}} < 4$

[117]. $\dfrac{9}{a+b+c} \leq \dfrac{2}{a+b} + \dfrac{2}{b+c} + \dfrac{2}{c+a} \leq \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}, (a,b,c > 0)$

[118]. $\dfrac{1}{2} \leq \dfrac{a^2}{a+b} + \dfrac{b^2}{b+c} + \dfrac{c^2}{c+d} + \dfrac{d^2}{d+a}, (a+b+c+d=1)$

[119]. $a^2b^2(a^2+b^2) \leq 2, (a+b=1, a,b > 0)$

[120]. $\sqrt[3]{\dfrac{a^2b^2(a+b)^2}{4}} \leq \dfrac{a^2+10ab+b^2}{12}, (ab > 0)$

[121]. **삼각형 세변:** $a,b,c \Rightarrow 1 \leq ab+bc+ca-abc \leq 1+\dfrac{1}{27}, (a+b+c=2)$

[122]. $a+b+c \leq \dfrac{a^3}{bc} + \dfrac{b^3}{ca} + \dfrac{c^3}{ab}, (a,b,c > 0)$

[123]. $a = x^{\frac{1}{12}}, b = x^{\frac{1}{4}}, c = x^{\frac{1}{6}} \Rightarrow 2^{1+c} \leq 2^a + 2^b, (x > 0)$

[124]. $a\sqrt{b}+b\sqrt{a} \le \dfrac{(a+b)^2}{2}+\dfrac{(a+b)}{4},\ (a,b>0)$

[125]. $a\sqrt{bc}+b\sqrt{ca}+c\sqrt{ab} \le \dfrac{(a+b+c)^2}{3},(a,b,c \ge 1)$

[126]. $a+b+c=1 \Rightarrow \sqrt{a+\dfrac{(b-c)^2}{4}}+\sqrt{b}+\sqrt{c} \le \sqrt{3}$

[127]. $x^2+y^2+z^2+2xyz=1 \ \Rightarrow\ 8xyz+4(x+y+z) \le 7,\ \left(x,y,z \in R^+\right)$

[128]. $a,b,c,d \ge 0,\ \ a^2+b^2+c^2+d^2=2$

$$\Rightarrow 2\left(a^3b+b^3c+c^3d+d^3a\right) \le a^4+b^4+c^4+d^4+1$$

[129]. $\triangle ABC \Rightarrow \sin\dfrac{3A}{2}+\sin\dfrac{3B}{2}+\sin\dfrac{3C}{2} \le \cos\dfrac{A-B}{2}+\cos\dfrac{B-C}{2}+\cos\dfrac{C-A}{2}$

[130]. $a+b+c=3 \Rightarrow \dfrac{(2a+b+c)^2}{2a^2+(b+c)^2}+\dfrac{(2b+c+a)^2}{2b^2+(c+a)^2}+\dfrac{(2c+a+b)^2}{2c^2+(a+b)^2} \le 8$

[131]. $abcd=1, a,b,c,d \in R^+ \Rightarrow \dfrac{1+ab}{1+a}+\dfrac{1+bc}{1+b}+\dfrac{1+cd}{1+c}+\dfrac{1+da}{1+d} \ge 4$

[132]. $0<\alpha_1,\alpha_2,\ \cdots,\alpha_n<\dfrac{\pi}{2}$

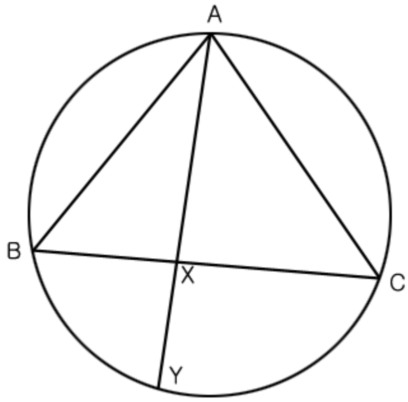
$$\Rightarrow\left(\frac{1}{\sin\alpha_1}+\cdots+\frac{1}{\sin\alpha_n}\right)\left(\frac{1}{\cos\alpha_1}+\cdots+\frac{1}{\cos\alpha_n}\right) \le 2\left(\frac{1}{\sin 2\alpha_1}+\cdots+\frac{1}{\sin 2\alpha_n}\right)^2$$

[133]. $x,y \in R^+, x^3+y^4 \le x^2+y^3 \Rightarrow x^3+y^3 \le 2$

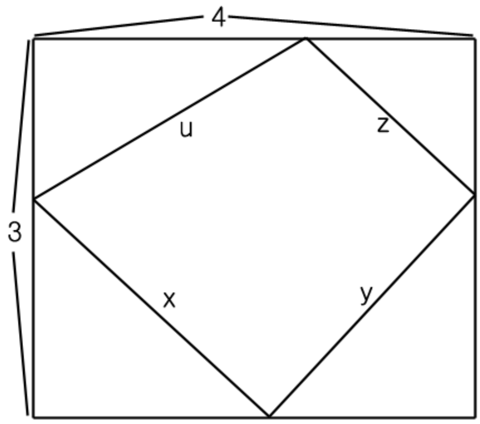
[134]. $0<x_i \le \dfrac{1}{2} \Rightarrow \left(\dfrac{n}{x_1+\cdots+x_n}-1\right)^n \le \left(\dfrac{1}{x_1}-1\right)\cdots\left(\dfrac{1}{x_n}-1\right)$

[135]. $a, b, c > 0, abc = 1 \Rightarrow \frac{1}{a+b+1} + \frac{1}{b+c+1} + \frac{1}{c+a+1} \leq 1$

[136]. $\frac{1}{AX} + \frac{1}{XY} \geq \frac{4}{BC}$



[137]. $25 \leq x^2 + y^2 + z^2 + u^2 \leq 50$



[138]. $a,b,c > 0 \Rightarrow \frac{1}{b(a+b)} + \frac{1}{c(b+c)} + \frac{1}{a(c+a)} \geq \frac{27}{2(a+b+c)^2}$

[139]. $s = x_1 + x_2 + \cdot\cdot + x_n,\ (x_i > 0)$

$$\Rightarrow (1+x_1)(1+x_2)\cdot\cdot(1+x_n) \leq 1 + s + \frac{s^2}{2!} + \cdot\cdot + \frac{s^n}{n!}$$

[140]. $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2},\ (a,b,c > 0)$

[141]. $abc=1 \Rightarrow \left(a-1+\frac{1}{b}\right)\left(b-1+\frac{1}{c}\right)\left(c-1+\frac{1}{a}\right) \le 1,\ (a,b,c>0)$

[142]. $a_i \ge 1 \Rightarrow \frac{1}{a_1+1}+\frac{1}{a_2+1}+\cdots+\frac{1}{a_n+1} \ge \frac{n}{\sqrt[n]{a_1 a_2 \cdots a_n}+1}$

[143]. $a \ge b \ge c,\ x+z \ge y,\ (x,y,z>0,\ k \in N)$

$\Rightarrow x^2(a-b)^k(a-c)^k+y^2(b-c)^k(b-a)^k+z^2(c-a)^k(c-b)^k \ge 0$

[144]. $abc=8,\ (a,b,c>0)$

$\Rightarrow \frac{a^2}{\sqrt{(a^3+1)(b^3+1)}}+\frac{b^2}{\sqrt{(b^3+1)(c^3+1)}}+\frac{c^2}{\sqrt{(c^3+1)(a^3+1)}} \ge \frac{4}{3}$

[145]. $a_1=0,\ |a_2|=|a_1+1|,\ \cdots,\ |a_n|=|a_{n-1}+1|$

$\Rightarrow \frac{a_1+a_2+\cdots+a_n}{n} \ge -\frac{1}{2}$

[146]. f : 연속, 단조증가, $f(0)=0,\ f(1)=1$

$\Rightarrow f\left(\frac{1}{10}\right)+f\left(\frac{2}{10}\right)+\cdots+f\left(\frac{9}{10}\right)+f^{-1}\left(\frac{1}{10}\right)+\cdots+f^{-1}\left(\frac{9}{10}\right) \le \frac{99}{10}$

[147]. $a,b,c \ge 2 \Rightarrow \log_{(a+b)} c+\log_{(b+c)} a+\log_{(c+a)} b \ge \frac{3}{2}$

[148]. $\left(1-\frac{1}{n}\right) < e\left(1-\frac{1}{n}\right)^n,\ (n>1)$

[149]. **삼각형 세변:** a,b,c, 삼각형넓이 : $A \Rightarrow a^2+b^2+c^2 \ge 4\sqrt{3}A$

[150]. $\sum_{k=1}^{n} kx_k \le \frac{n(n-1)}{2}+\sum_{k=1}^{n} x_k^k,\ (x_i>0)$

[151]. $\dfrac{x+y}{x^2-xy+y^2} \le \dfrac{2\sqrt{2}}{\sqrt{x^2+y^2}}, \quad (x+y>0)$

[152]. $n \le m, (n,m \in N) \Rightarrow 2^n(n!) \le \dfrac{(m+n)!}{(m-n)!} \le \left(m^2+m\right)^n$

[153]. $\sqrt{x}+\sqrt{y}+\sqrt{z}=1 \Rightarrow \dfrac{x^2+yz}{\sqrt{2x^2(y+z)}}+\dfrac{y^2+zx}{\sqrt{2y^2(z+x)}}+\dfrac{z^2+xy}{\sqrt{2z^2(x+y)}} \ge 1$

[154]. $0 < x_i < \dfrac{1}{2} \Rightarrow \dfrac{\prod_{i=1}^{n} x_i}{\left(\sum_{i=1}^{n} x_i\right)^n} \le \dfrac{\prod_{i=1}^{n}(1-x_i)}{\left(\sum_{i=1}^{n}(1-x_i)\right)^n}$

[155]. $n > 1 \Rightarrow \dfrac{1}{2} < \dfrac{1}{n^2+1}+\dfrac{2}{n^2+2}+\cdot\cdot+\dfrac{n}{n^2+n} < \dfrac{1}{2}+\dfrac{1}{2n}$

[156]. $a,b,c \in R^+ \Rightarrow \left(\ln\dfrac{ab}{c}\right)^2+\left(\ln\dfrac{bc}{a}\right)^2+\left(\ln\dfrac{ca}{b}\right)^2+\dfrac{3}{4} \ge \ln(abc)$

[157]. $x_1 \ge x_2 \ge .. \ge 0, \ \sum_{i=1}^{n} x_i \le 400, \ \sum_{i=1}^{n} x_i^2 \ge 10^4$

$\Rightarrow \sqrt{x_1}+\sqrt{x_2} \ge 10, \ (n \ge 2)$

[158]. $\min\{(a-b)^2, (b-c)^2, (c-a)^2\} \le \dfrac{1}{2}\left[a^2+b^2+c^2\right]$

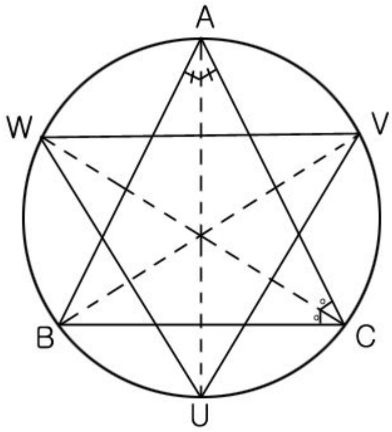
[159]. $\min\{(a-b)^2,(a-c)^2,(a-d)^2,(b-c)^2,(b-d)^2,(c-d)^2\} \le \dfrac{1}{5}\left[a^2+b^2+c^2+d^2\right]$

[160]. $a,b,c,d>0 \Rightarrow \sqrt{\frac{a}{b+c}}+\sqrt{\frac{b}{c+d}}+\sqrt{\frac{c}{d+a}}+\sqrt{\frac{d}{a+b}}>2$

[161]. $y \geq 0,\ y(y+1) \leq (x+1)^2 \Rightarrow y(y-1) \leq x^2$

[162]. $av-bu=1 \Rightarrow a^2+u^2+b^2+v^2+au+bv \geq \sqrt{3}$

[163]. $\triangle ABC$넓이 $\leq$ $\triangle UVW$넓이



[164]. $z,w \in C \Rightarrow (|z|+|w|)\left|\frac{z}{|z|}+\frac{w}{|w|}\right| \leq 2|z+w|$

[165]. $\sqrt{x^2-xy+y^2}+\sqrt{y^2-yz+z^2} \geq \sqrt{x^2+xz+z^2}$, $(x,y,z>0)$

[166]. $a_1=\frac{1}{2},\ a_{n+1}=\frac{a_n^2}{a_n^2-a_n+1} \Rightarrow a_1+a_2+\cdots+a_n<1$

[167]. $f''(x)>0,\ x_1<x_2<\dots<x_n$

$\Rightarrow x_1f(x_2)+x_2f(x_3)+\cdots+x_nf(x_1) \geq x_2f(x_1)+x_3f(x_2)+\cdots+x_1f(x_n)$

[168]. $\{a_n\}$: 등차수열, $\{b_n\}$: 등비수열, $a_1=b_1,\ a_n=b_n$

$\Rightarrow a_1+a_2+\cdots+a_n \geq b_1+b_2+\cdots+b_n$

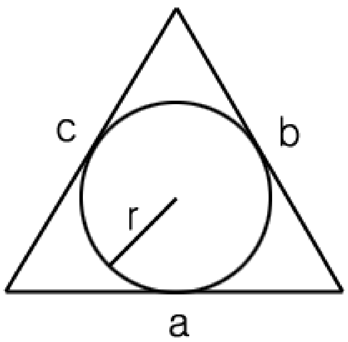
[169]. $a+b+c=abc,\ (a,b,c>0)$

$$\Rightarrow \frac{1}{\sqrt{1+a^2}}+\frac{1}{\sqrt{1+b^2}}+\frac{1}{\sqrt{1+c^2}} \le \frac{3}{2}$$

[170]. $1<a<b<c \Rightarrow \log_a(\log_a b)+\log_b(\log_b c)+\log_c(\log_c a)>0$

[171]. $\frac{x}{x+\sqrt{(x+y)(x+z)}}+\frac{y}{y+\sqrt{(x+y)(y+z)}}+\frac{z}{z+\sqrt{(x+z)(y+z)}} \le 1, (x,y,z>0)$

[172]. $s=\frac{a+b+c}{2} \Rightarrow \frac{1}{(s-a)^2}+\frac{1}{(s-b)^2}+\frac{1}{(s-c)^2} \ge \frac{1}{r^2}$



[173]. $a+b+c=1,\ (a,b,c>0) \Rightarrow \frac{a^3}{a^2+b^2}+\frac{b^3}{b^2+c^2}+\frac{c^3}{c^2+a^2} \ge \frac{1}{2}$

[174]. $a+b=1, (a,b>0) \Rightarrow \left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2 \ge \frac{25}{2}$

[175]. $a,b,c>0, a<bc, 1+a^3=b^3+c^3 \Rightarrow 1+a<b+c$

[176]. $a,b,x,y>0,\ a^5+b^5 \le 1,\ x^5+y^5 \le 1$

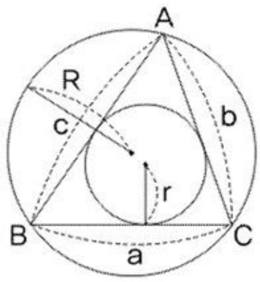
$$\Rightarrow a^2x^3+b^2y^3 \le 1$$

[177]. **삼각형 세변:** $a,b,c \Rightarrow \frac{1}{3} \le \frac{a^2+b^2+c^2}{(a+b+c)^2} < \frac{1}{2}$

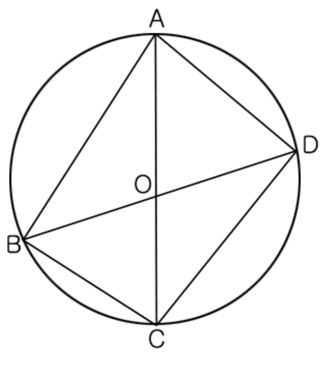
[178]. $m,n>0, |f(m+n)-f(n)| \le \frac{n}{m} \Rightarrow \sum_{i=1}^{k} |f(2^k)-f(2^i)| \le \frac{k(k-1)}{2}$

[179]. $a_n, b_n > 0,\ a_{n+1} = a_n + \frac{1}{b_n},\ b_{n+1} = b_n + \frac{1}{a_n} \ \Rightarrow\ a_{50} + b_{50} > 20$

[180]. $\frac{r}{2R} \le \frac{abc}{\sqrt{2(a^2+b^2)(b^2+c^2)(c^2+a^2)}}$



[181]. $\frac{AB}{CD} + \frac{CD}{AB} + \frac{BC}{AD} + \frac{AD}{BC} \le \frac{OA}{OC} + \frac{OC}{OA} + \frac{OB}{OD} + \frac{OD}{OB}$



[182]. $a,b,c>0 \Rightarrow 9(a^3+b^3+c^3) \ge (a+b+c)^3$

[183]. $0 < \alpha,\beta,\gamma < \frac{\pi}{2}, \alpha+\beta+\gamma = \frac{\pi}{2} \Rightarrow \frac{1-\sin\alpha}{1+\sin\alpha} + \frac{1-\sin\beta}{1+\sin\beta} + \frac{1-\sin\gamma}{1+\sin\gamma} \ge 1$

[184]. $f(0)=0, |f'(x)| \le \dfrac{1}{1+x} \Rightarrow \displaystyle\int_0^{e-1} \{f(x)\}^2 dx \le e-2$

[185]. $1 \le \displaystyle\int_0^{\frac{\pi}{2}} \sqrt{1-\sin^3 x}\, dx \le \dfrac{1}{2}\{\sqrt{2}+\ln(1+\sqrt{2})\}$

[186]. $n>2 \Rightarrow \dfrac{1}{2} < \displaystyle\int_0^{\frac{1}{2}} \dfrac{dx}{\sqrt{1-x^n}} < \dfrac{\pi}{6}$

[187]. $\dfrac{1}{2}-\dfrac{\pi}{48}-\dfrac{7\sqrt{3}}{32} \le \displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \ln(1+\cos 2x)\, dx \le \dfrac{1}{4}(2-\sqrt{3})$

[188]. $f(x)=1-\sin x,\ g(x)=\displaystyle\int_0^x (x-t)f(t)\, dt$

$\Rightarrow g(x+y)+g(x-y) \ge 2g(x)$

[189]. $1^x+2^x+6^x+12^x \ge 4^x+8^x+9^x$

[190]. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \dfrac{2x}{\sin x}\, dx \ge \dfrac{2\pi^2}{9}$

[191]. $\dfrac{\pi}{2}-1 < \displaystyle\int_0^1 e^{-2x^2}\, dx$

[192]. $a,b,c \in R^+ \Rightarrow (a+b+c)\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right) \le \left(\dfrac{a}{b}+\dfrac{b}{c}+\dfrac{c}{a}\right)^2$

[193]. $x>y>0,\ a>b>0 \Rightarrow (x^b-y^b)^a < (x^a-y^a)^b$

[194]. $\ln(1+x) \le \int_x^{x^2} \frac{1}{\ln t} dt \le \ln\left|\frac{(1+x)(3-x)}{3-x^2}\right|$

[195]. $\frac{1-x^2}{2} \le \int_x^1 f(t)dt \Rightarrow \frac{1}{3} \le \int_0^1 f^2(t)\,dt$

[196]. $\left(\int_a^b f(x)dx\right)\left(\int_a^b \sqrt{f^4(x)+1}\,dx\right) \le \frac{\left[\int_a^b \left(f^2(x)+1\right)dx\right]^2}{2\sqrt{2}}$

[197]. $a_1, a_2, .., a_n \in R \Rightarrow \sum_{i,j=1}^{n} \frac{a_i a_j}{i+j-1} \ge 0$

[198]. $a, b, c \ge 0,\ a+b+c=1 \Rightarrow a^2+b^2+c^2 \ge 4(ab+bc+ca)-1$

[199]. $\frac{3}{4}\left(\int_0^1 f(x)dx\right)^2 \le \int_0^1 f^2(x^2)\,dx$

[200]. $\int_0^1 f(x)dx = \int_0^1 x f(x)dx = 1 \Rightarrow \int_0^1 f^2(x)dx \ge 4$

[201]. $f(x) > 0 \Rightarrow \int_0^1 f(x)dx \cdot \int_0^1 f^2(x)dx \le \int_0^1 f^3(x)dx$

[202]. $a_1 = \sqrt{6},\ a_{n+1} = \sqrt{6+a_n} \Rightarrow \frac{3-\sqrt{6}}{6^{n-1}} \le 3-a_n < \frac{3}{5^n}$

[203]. $f(x)$:실근을 갖는 다항식 $\Rightarrow f(x)f''(x) \le f'(x)^2$

[204]. $f(x)$: 연속, $f(a)=f(b)=0 \Rightarrow \left|\int_a^b f(x)dx\right| \le \dfrac{(b-a)^2}{4}\max|f'(x)|$

[205]. $\sin(\cos x) < \cos(\sin x)$

[206]. $\forall x \in R \Rightarrow x^4 + x^3 + \dfrac{1}{2}x^2 + \dfrac{1}{6}x + \dfrac{1}{24} > 0$

[207]. $\forall n \in N \Rightarrow 2 \le \left(1+\dfrac{1}{n}\right)^n < 3$

[208]. $1 \le m \Rightarrow \left(\dfrac{a_1 + a_2 + \cdot\cdot\cdot + a_n}{n}\right)^m \le \dfrac{a_1^m + a_2^m + \cdot\cdot\cdot + a_n^m}{n}$

[209]. $1 \le m \Rightarrow \dfrac{\sqrt[m]{a_1} + \cdot\cdot\cdot + \sqrt[m]{a_n}}{n} \le \sqrt[m]{\dfrac{a_1 + a_2 + \cdot\cdot\cdot + a_n}{n}}$

[210]. $a, b, c > 0, \quad a+b+c=1$

$$\Rightarrow \frac{\sqrt{a^2+abc}}{c+ab} + \frac{\sqrt{b^2+abc}}{a+bc} + \frac{\sqrt{c^2+abc}}{b+ca} \le \frac{1}{2\sqrt{abc}}$$

[211]. $0 \le x, y \le \dfrac{\pi}{2} \Rightarrow \sqrt[5]{\cos^4 x \sin^6 y} + \sqrt[5]{\sin^4 x \cos^6 y} \le 1$

[212]. $\forall x \in R \Rightarrow -1 < \sqrt{x^2+x+1} - \sqrt{x^2-x+1} < 1$

[213]. $a, b, c \in N \Rightarrow \dfrac{a+b+c}{3} \le a^{\frac{a}{a+b+c}} b^{\frac{b}{a+b+c}} c^{\frac{c}{a+b+c}}$

[214]. $\cos 1 \bullet \cos 2 \bullet \bullet \bullet \cos 2^{n-1} \bullet \cos 2^{n} < \frac{\sqrt{2}}{2^{n+1}}$

[215]. $a,b,c>0, abc=1 \Rightarrow 1 < \frac{1}{\sqrt{1+2a}} + \frac{1}{\sqrt{1+2b}} + \frac{1}{\sqrt{1+2c}}$

[216]. $\forall\, x,y \in R^{+} \Rightarrow 3xy \le 2x\sqrt{x} + y^{3}$

[217]. $x,y,z \ge 0 \Rightarrow x^{p}(x-y)(x-z) + y^{p}(y-z)(y-x) + z^{p}(z-x)(z-y) \ge 0$

[218]. $a,b,c>0, a+2b+3c=1 \Rightarrow (1-a)(1-b)^{2}(1-c)^{3} \ge 5^{6}ab^{2}c^{3}$

[219]. $x,y>0,\ x,\ y \ne 1 \Rightarrow \left|\log_{x}y + \log_{y}x\right| \ge 2$

[220]. $\frac{1}{2} \bullet \frac{3}{4} \bullet \frac{5}{6} \bullet \bullet \frac{99}{100} < \frac{1}{\sqrt{101}}$

[221]. $a,b,c>0 \Rightarrow 1 < \frac{a}{a+b} + \frac{b}{b+c} + \frac{c}{c+a} < 2$

[222]. $2(xy+yz+zx) \le x^{2}\left(\frac{b+c}{a}\right) + y^{2}\left(\frac{a+c}{b}\right) + z^{2}\left(\frac{a+b}{c}\right)$